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The value of using inductive and deductive methods in instructing a mathematical principle and foreign language alphabet are tested with sample groups of sixth and eighth grade students in two separate educational experiments. In both cases, the instructional methods used were--(1) instruction in principle before application to examples, (2) examples followed by principle clarification and supplemented by further examples, and (3) examples only. Although students taught by the first method tended to learn the principles best, the participants in the third groups were superior in the areas of retention and transfer. The performance level of the second groups lay between those of the other two. For a related document see ED 022 392. (AF)

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THE VALUE OF EXTERNAL DIRECTION AND INDIVIDUAL DISCOVERY IN LEARNING SITUATIONS

I. The learning of a mathematical principle

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Scand. J. Psychol., 1968, 9, 241–247

THE VALUE OF EXTERNAL DIRECTION AND INDIVIDUAL DISCOVERY IN LEARNING SITUATIONS

I. *The learning of a mathematical principle*

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WERDELIN, I. The value of external direction and individual discovery in learning situations. I. The learning of a mathematical principle. *Scand. J. Psychol.*, 1968, 9, 241–247.—Three samples (A, B, C), selected at random from seven sixth grade classes, were instructed differently how to use a mathematical principle. A was told the principle and applied it on examples; B was given most examples first, then told the principle, and given additional examples; C was given examples only. A learned the principle better, as measured by a test given immediately after the experiment. C was comparatively better as to retention and transfer. B was between A and C.

The aim of this study was to compare the so-called discovery method of teaching for the generalization of a rule or principle, where the students are to discover this individually from a number of examples, with the more common method where the principle is presented and applied. In psychological literature we find a number of reports of studies on how principles should be taught in order to facilitate learning, retention and transfer. Typically they compare a situation, where students are given a large number of examples from which they can deduce the principle, with a situation, where the teacher presents the principle and works out one or a few examples but the students are passive.

Of the studies which have influenced the present study we want to refer to a few. Two of these stress the importance of the direction and guidance given by the teacher. Craig (1953, 1956) compared a situation where a group of college students were given directions as to the principle with a situation where they were only given the instructions that there was a principle and that one of five items given did not belong. He found that the directed group solved significantly more such problems, but that there was little difference as to transfer and retention.

Kittell (1957), who used Craig's material but had a group of sixth graders, distinguished between a 'maximal direction' group (the experimenter explained the principle and worked out the answers), an 'intermediate direction' group (the experimenter started the principle), and a 'minimal direction' group (the subjects were told that one item in each group did not belong with the others). The second group was found significantly superior to the others, while the minimal direction group was the inferior one. However, due to the fact that the problems presented were very difficult to the subjects, it is probable that these had to be told about the principle in order to discover that there was one.

In a well-known experiment Hendrix (1950) distinguished between three methods of teaching a mathematical principle: one where the principle is told and illustrated, one where the students have to discover it from examples and verbalize it, and one where they dis-

cover it and leave it unverbalized. In her experiment the subjects were evidently sufficiently instructed so that they learned the principle to perfection. After two weeks they were given a test where they had to recognize the application of the principle when solving certain problems. It was found that the last-mentioned method was superior to the second which was superior to the first. What Hendrix studied was not properly transfer in general but the effect of the various methods on the ability to recognize an instance of the principle taught. This is, of course, very important. Unfortunately, there is no indication of a statistical proof of the hypothesis in Hendrix' report, and therefore we cannot unhesitantly accept the results.

A study by Haslerud & Meyers (1958) illustrates very well the problem of the discovery method. University students were given two types of experience in deciphering codes: (1) They were provided with specific directions for deciphering the codes printed above the problems. (2) No direction was given. A different code was used in each problem. The subjects served as their own control and solved an approximately equal number of each kind of items. They did significantly better on the items where the principle was given, which might be expected. After one week a multiple-choice transfer test consisting of sentences illustrating the coding principles of the first test was given. It was found that the scores were significantly increased for those problems which had formerly been derived, while they were decreased for those problems where the rule had been given. Haslerud & Meyers found that their results indicated that the discovery method with individually derived principles is superior in transfer value to the method where the subjects were given specific directions for deciphering the codes.

That the results of these studies are controversial is obvious. We should not expect complete agreement. People might differ with respect to the type of teaching and instruction they would profit from. Therefore we would have to carry out studies on several samples, which differ with respect to variables like intelligence, age, experience, etc. But it is also most likely that there are differences between different materials and different types of generalizations. We must also distinguish between various results of the teaching situation: learning, retention, and various aspects of transfer. To arrive at a universal law must be our final aim, but to do this we must study the problem systematically.

In the following the author will present the results of the first of a series of studies on this problem. It has partly the character of a preliminary experiment, which should pave the way for more extensive studies in this important field, which also concerns teaching methods in the class-room, ways of writing text books, etc. with a view to efficient concept learning.

EXPERIMENT

The 211 students of 7 sixth grade classes were randomly distributed on three experimental groups, which will be called samples A, B, and C. As some of the students were absent due to illness, the three samples numbered 58, 63, and 57 subjects, respectively. Though this can hardly be judged necessary, a check was made that the three groups did not differ significantly in important variables like intelligence, verbal ability, marks, etc.

In the experiment we wanted to teach the left distributive principle of multiplication over addition. A comparison was made between the three methods of teaching. Due to the nature

of the mathematics instruction given in Swedish elementary schools, the subjects had no previous experience of the distributive principle.

Sample A was given a booklet with the following contents: The distributive principle in the form used here

$$a \times b + a \times c = a(b + c),$$

was thoroughly explained in words and by means of 12 solved examples. After the subjects had read the explanations, they were allowed to practice the principle on another 78 examples. Sample B was given a booklet where there were first 74 examples, which the subjects are told to solve, then the explanation of the principle and 3 solved examples, and then another 13 examples on which to practice the application of the principle. Sample C was given the 90 examples to solve but no explanation.

The 90 examples, which were thus common to the three samples, were selected so that the subjects should be able to derive the principle from them. They range in difficulty from e.g.

$$20 \times 2 + 20 \times 8 =$$

via

$$32 \times 2 + 32 \times 48 =$$

to e.g.

$$94 \times 36.95 + 94 \times 23.05 = .$$

It was evident that the use of the principle did not become clear to all the subjets in either group. The subjects were told to hand in all computations, and from the way they solved the tasks it was possible to see whether they used the principle or not.

To measure to what extent they had profited from the training, they were given four final tests of 8 items each. Each test took 6 minutes, which enabled those subjects who had understood the principle to solve all the items, but those who had not understood it could only solve a few.

Test 1 contained items of the same type as those in the booklet: They measured the direct application of the left distributive principle. Test 2 contained similar items but whith the right distributive principle:

$$6 \times 64 + 44 \times 64 = .$$

Test 3 contained items where the left distributive principle was applied to items with three terms:

$$37 \times 50 + 37 \times 40 + 37 \times 10 =,$$

and test 4 items where it was applied to subtraction:

$$637 \times 293 - 637 \times 193 = .$$

In this way we wanted to measure to what extent the application of the principle is transferred to other types of problems.

To measure retention four tests were given after exactly two weeks. These tests, which were called tests I–IV, paralleled tests 1–4, respectively.

It should be noted that the instructions given were entirely written; the teachers took no part in the experiment.

RESULTS

The data from each subject consisted of two types of scores: (1) The number of correctly solved items, ranging from 0 to 8, on each of the tests. (2) Information about whether the subject correctly used the distributive principle in solving the problems of each of the tests.

If the principle had been used correctly in the test, we called this a *positive* score, if it had not been used or had been used correctly in only some items of the test, we called this a *negative* score. A positive score thus indicated that the subject had understood the principle.

TABLE 1. *Proportions of positive scores and medians.*

Test and aspect	Group A	B	C	Test and aspect	Group A	B	C
1				I			
Proportion pos. scores	0.88	0.68	0.24	Proportion pos. scores	0.67	0.49	0.45
Median	6.88	6.18	1.89	Median	6.08	4.33	3.71
2				II			
Proportion pos. scores	0.67	0.51	0.21	Proportion pos. scores	0.69	0.57	0.46
Median	6.30	4.88	2.39	Median	7.12	6.20	4.25
3				III			
Proportion pos. scores	0.81	0.57	0.21	Proportion pos. scores	0.74	0.64	0.52
Median	7.10	5.62	2.27	Median	7.20	6.78	6.00
4				IV			
Proportion pos. scores	0.38	0.38	0.15	Proportion pos. scores	0.60	0.49	0.46
Median	3.00	2.63	2.21	Median	6.50	4.40	4.88

The relationship between the two types of scores was, of course, very high: A positive score also meant that there were many correctly solved items. (In the twenty-four cases with the three samples and the eight tests, the medians were always close to 7.) A negative score nearly always meant that there were few correctly solved items. (The medians were close to 1.)

As the distribution of the scores on each test is composed of two different distributions, most statistical parameters are inapplicable. Owing to the extremely U-shaped distribution curves, measures like the median do not give much information about the distribution, either: Slight variations as to the composition of the samples may mean large variations as to the median etc.

Table 1 gives the proportions of positive scores and the medians for the eight tests and the three samples. As can be seen there are quite pronounced differences between the three samples.

The probabilities that these differences can be caused by sampling errors only are given in Table 2. In the case of the proportions, the common parametric test for differences between proportions was used. To see whether there were differences as to the shapes of the distribution curves, we used the Kolmogorov-Smirnov two-sample test.

Sample A was superior to sample B in most cases, but the superiority is significant only with respect to test 3 and the proportions of positive scores in tests 1 and I. In all other cases the differences were clearly insignificant. These two samples seemed superior to sample

TABLE 2. *Probabilities of differences between groups.*

Test and aspect	Groups compared		
	A and B	B and C	A and C
1			
Proportions pos. scores	$0.05 > p > 0.025$	$p < 0.001$	$p < 0.001$
Distribution	$p > 0.10$	$p < 0.001$	$p < 0.001$
2			
Proportion pos. scores	$p > 0.10$	$p < 0.001$	$p < 0.001$
Distribution	$p > 0.10$	$0.025 > p > 0.01$	$p < 0.001$
3			
Proportion pos. scores	$0.01 > p > 0.005$	$p < 0.001$	$p < 0.001$
Distribution	$0.10 > p > 0.05$	$0.005 > p > 0.001$	$p < 0.001$
4			
Proportion pos. scores	$p > 0.10$	$0.01 > p > 0.005$	$0.01 > p > 0.005$
Distribution	$p > 0.10$	$0.10 > p > 0.05$	$0.10 > p > 0.05$
I			
Proportion pos. scores	$0.10 > p > 0.05$	$p > 0.10$	$0.025 > p > 0.01$
Distribution	$p > 0.10$	$p > 0.10$	$0.05 > p > 0.10$
II			
Proportion pos. scores	$p > 0.10$	$p > p.10$	$0.05 > p > 0.025$
Distribution	$p > 0.10$	$p > 0.10$	$p > 0.10$
III			
Proportion pos. scores	$p > 0.10$	$p > 0.10$	$0.05 > p > 0.025$
Distribution	$p > 0.10$	$p > 0.10$	$p > 0.10$
IV			
Proportion pos. scores	$p > 0.10$	$p > 0.10$	$p > 0.10$
Distribution	$p > 0.10$	$p > 0.10$	$p > 0.10$

C, and the differences were on a significant level with respect to tests 1–4 and in the case of sample A also with respect to test I and the proportions of positive scores in tests II and III.

SUMMARY AND CONCLUSIONS

Data thus indicate that there was a significant superiority of sample A to the other samples in the tests 1 and 3, which measured the ability of the subjects to apply to left distributive principle immediately after the training period. In tests 2 and 4 it was superior to

TABLE 3. *Directions of and probabilities for differences between proportions of positive scores in certain tests.*

Tests compared	Group A	Group B	Group C
1 and I	Decrease $p<0.001$	Decrease $0.005>p>0.001$	Increase $p<0.001$
2 and II	$p>0.10$	$p>0.10$	Increase $p<0.001$
3 and III	$p>0.10$	$p>0.10$	Increase $p<0.001$
4 and IV	Increase $p<0.001$	$p>0.10$	Increase $p<0.001$
1 and 2	Decrease $0.005>p>0.001$	Decrease $0.01>p>0.005$	$p>0.10$
1 and 3	$p>0.10$	$p>0.10$	$p>0.10$
1 and 4	Decrease $p<0.001$	Decrease $p<0.001$	Decrease $0.10>p>0.05$
I and II	$p>0.10$	Increase $0.10>p>0.05$	$p>0.10$
I and III	$p>0.10$	Increase $0.005>p>0.001$	Increase $0.10>p>0.05$
I and IV	$p>0.10$	$p>0.10$	$p>0.10$

The word *decrease* means that the first-mentioned test shows a significantly larger number of positive scores than the last-mentioned one.

sample C only. These two tests measured the transfer to somewhat different situations. With respect to tests I–IV, which measured retention after two weeks, the superiority of sample A over sample B and of the latter over sample C was further diminished the more the tests deviated from test I.

This was accompanied by changes in the proportions of positive scores and in the shapes of the distribution curves for the different groups. While sample A showed its best performances in tests 1 and 3, which, as we said, measured the ability to apply the principle learned immediately, sample B and particularly sample C seemed to perform comparatively better and better the more we leave the original learning situation in time and with respect to stimulus similarity.

While available statistical methods do not allow us to compare the distributions of the different tests for the same group, the proportions of positive scores are directly comparable. In Table 3 we give the probability figures (computed by means of the Sign test) for differences between test I and the other tests given in the first experiment, between test I and

the other retention tests, and between tests 1 and I, tests 2 and II, tests 3 and III, and tests 4 and IV.

Data indicate that sample A was superior to sample B and this to sample C in learning the principle and applying it immediately. The subjects profited from having the rule explained to them as early as possible. But with regard to the retention of the principle the situation is somewaht different. All groups showed high ability of recalling it again, but sample C increased its performance significantly. They even became almost as good as the other two groups. Also with respect to their ability of transfer there were differences between the groups. While many of the subjects of sample A can apply the principle in a limited sphere only, a larger proportion of the subjects of samples B and C who understood the principle were able to apply it in somewhat different situations.

More subjects of sample A learned the principle, but some of them either forgot it or were unable to apply it in new situations, while fewer subjects of sample C learned it, but they showed stable or increased ability to apply it after some time or in new situations. Sample B is between the other groups. This must be due to the three methods of instruction. While we are still far from a final explanation, the present author would like to state a hypothesis, which may explain this phenomenon.

The learning of a mathematical principle should be seen as an example of concept formation. A concept can get meaning on several levels or in several dimensions. (See e.g. Van Engen, 1953.) Of particular importance are the syntactic dimension (the symbol, word, formula or whatever represents the concept has meaning in so far as it can be used in relation to other concepts), and the semantic dimension (the symbol etc. has a referent; there is something behind it). In the syntactic dimension (or on the syntactic level) we can apply the symbol etc.; it is functional. First on the semantic level do we get full understanding, however.

It is most likely that some of the subjects who are told the principle first, concentrate on the syntactic dimension. They acquire a functional ability of handling the tasks. To the subjects who discover the principle from the examples, the semantic dimension is probably more in the focus. It is probable that one's having stressed the semantic dimension is more favorable to retention and transfer. Even if this has not been proved, it seems a reasonable hypothesis worth studying.

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II. The learning of a foreign alphabet

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II. *The learning of a foreign alphabet*

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WERDELIN, I. The value of external direction and individual discovery in learning situations. II. The learning of a foreign alphabet. *Scand J. Psychol.*, 1968, 9, 248–251.—Three samples (A, B, C), selected at random from seven eighth grade classes and matched with respect to scholastic achievement, line of study, and sex, were taught a foreign alphabet differently. A was told the principles of the alphabet and applied it on examples; B was given most examples first, then told the principles, and given additional examples; C was given the examples only. There was a clear tendency that A learned the principles best as measured by the ability to apply them immediately but C was comparatively better on tests which measured retention and transfer.

By means of this study we wanted to compare the so-called discovery method of teaching the application of a principle, where the students are allowed to discover this individually from given examples, with the method where they are told how to use it. Previous investigations have not given final answers to the questions which of these methods is the best one in different school situations. Some authors have found that the directions given by the teachers are essential to learning (cf. e.g. Craig, 1953, 1956; Kittell, 1957), while others stress the importance of the discovery method to transfer (see Hendrix, 1950; Haslerud & Meyers, 1958).

In a previous study (Werdelin, 1962) three methods of learning a mathematical principle, were compared, one where the subjects were told the principle and applied it on a set of examples, one where they were first given some of the examples, then the principle, and finally the rest of the examples, and one where they were given the examples only. It was found that the subjects who were instructed according to the first method learned the principle best and were superior to the other groups when given a test immediately after the learning period, but the subjects who were given their instructions according to the third method showed comparatively better ability of transferring their knowledge and better retention.

In this study we want to compare the same methods of instruction on a new material. It is quite possible that we will get different results for different materials and different groups of subjects.

EXPERIMENT

From the students of seven eighth grade classes were put together 58 groups of three students each matched with respect to sex, line of study, and marks (total for all subjects studied). In the common way they were randomly distributed on three samples of 58 subjects each, called samples A, B, and C.

In the experiment we wanted to teach the use of the Arabic alphabet, which seems suitable for this purpose, because it is based on certain principles different from those of ours. The three principles treated in the experiment are:

(I) The letters are written from right to left.
(II) Different forms of the letters are used in different parts of the words.
(III) Short vowels are not represented by letters.

TABLE 1. *Proportions of correctly used principles and medians.*

Test and aspect	Sample A	Sample B	Sample C	Test and aspect	Sample A	Sample B	Sample C
Test 1				*Test 3*			
Medians	9.00	5.50	8.00	Medians	5.25	4.93	5.00
Proportions				Proportions			
Principle I	0.90	0.81	0.77	Principle I	0.79	0.78	0.86
Principle III	0.67	0.60	0.62	Principle III	0.59	0.43	0.52
Test 2				*Test 4*			
Medians	6.50	5.24	5.30	Medians	6.50	6.50	8.00
Proportions				Proportions			
Principle I	0.79	0.66	0.66	Principle I	0.71	0.74	0.83
Principle II	0.12	0.16	0.05	Principle II	0.10	0.10	0.03
Principle III	0.59	0.57	0.66	Principle III	0.57	0.48	0.57

Sample A was given a booklet to read. The principles were given one at a time, followed by a number of solved examples and a number of exercises to be solved by the subjects. On the final page there were a number of 'mixed' examples. The instructions concerned how to pronounce the words which were written in the foreign alphabet. Sample B was given a booklet which first contained the examples to the individual principles and exercises to these; then the principles were given, and finally the subjects could apply them to the 'mixed' examples. Sample C was given a booklet with the examples and exercises only.

The instructions were entirely written. In addition the subjects were given a sheet of paper with the Arabic characters used and another with the correct solutions to the exercises.

The ability of the subjects to read the alphabet was measured by means of a written test of 12 items, test 1, where they were told to write the pronunciation of certain words. When scoring we noticed whether the subjects had applied principles I and III correctly; we also found the number of words that were correctly transcribed. This gave us three scores for each subject. To measure transfer to a somewhat similar situation the subjects were also given a test, (2), where they were asked to transcribe words written in ordinary writing to the Arabic alphabet. Here we noticed whether the three principles were used corrctly, and we also found the number of words where the correct characters were used and at least principles I and III correctly applied.

After two weeks the testing was repeated with two parallel tests, scored in the same way. The latter tests are called tests 3 and 4.

RESULTS

The results are presented in Table 1. For each test the proportions of cases are reported where each principle has been correctly applied and the medians of the numbers of correctly solved items. As can be seen the three samples are rather close to one another. Only one significant difference can be found: principle I in test 1, sample A, was significantly superior to the other samples combined ($0.10 > p > 0.05$). On the whole there is a strong tendency for sample A to be superior in test 1, i.e. in the text which measures the ability to apply immediately what the subjects have learned.

TABLE 2. *Directions of and probabilities of differences between proportions of positive scores in the tests.*

Tests and principles compared	Sample A	Sample B	Sample C
1:I and 2:I	Decrease $0.10 > p > 0.05$	Decrease $0.05 > p > 0.01$	Decrease $0.10 > p > 0.05$
1:I and 3:I	Decrease $0.10 > p > 0.05$	Decrease $p > 0.10$	Decrease $p > 0.10$
1:I and 4:I	Decrease $0.005 > p > 0.001$	Decrease $p > 0.10$	Increase $p > 0.10$
1:III and 2:III	Decrease $p > 0.10$	Decrease $0.01 > p > 0.005$	Increase $p > 0.10$
1:III and 3:III	Decrease $p > 0.10$	Decrease $p > 0.10$	Increase $p > 0.10$
1:III and 4:III	Decrease $p > 0.10$	Decrease $p > 0.10$	Decrease $p > 0.10$
2:I and 4:I	Decrease $p > 0.10$	Decrease $p > 0.10$	Increase $0.05 > p > 0.01$
2:III and 4:III	$p = 1$	Decrease $0.05 > p > 0.01$	Decrease $p > 0.10$

The word *decrease* means that the first-mentioned test and principle shows significantly larger proportions of positive scores than the last-mentioned ones.

When we study transfer and retention the picture is different, however. In the case of principle I we find that the ability of sample A decreases the more the test differs from test 1 as to time and stimulus similarity, while the proportions of correct application of the principle by sample C is higher in tests 3 and 4 than in tests 1 and 2; this difference between the groups is clearly significant, as can be seen from Table 2. With respect to principle III we find the same tendency, but the differences we find are not significant. For principle II we find so low proportions of correct application that we cannot draw any conclusions.

DISCUSSION

Data are not in favor of any particular method of instruction. There is a tendency that the students who are told the principles are somewhat superior to the other groups, but, on the other hand, it is evident that students who discover the principles from examples are

comparatively more able in situations which involve transfer and retention over two weeks. The aim of educational research must be to look for a general law or rule, but we are still far from it. From what we can find from this experiment, there is not much difference between the methods applied to this material. However, there are indications that there are differences between materials. (cf. Werdelin, 1968).

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